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REVIEW ARTICLE

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Analysis and control of a Depression Model

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ABSTRACT

Millions of people are affected by depression. It is important to understand the progression dynamics of this disease to be able to minimize the damage that is caused by it. This article provides a mathematical framework to develop strategies to control depression. Bifurcation analysis is a powerful mathematical tool used to deal with the nonlinear dynamics of any process. Several factors must be considered, and multiple objectives must be met simultaneously. Bifurcation analysis and multiobjective nonlinear model predictive control (MNLMPC) calculations are performed on a depression. The MATLAB program MATCONT was used to perform the bifurcation analysis. The MNLMPC calculations were performed using the optimization language PYOMO in conjunction with the state-of-the-art global optimization solvers IPOPT and BARON. The bifurcation analysis revealed the existence of a branch point. The branch point is beneficial because it enables the multiobjective nonlinear model predictive control calculations to converge to the Utopia point which is the most beneficial solution. A combination of bifurcation analysis and multiobjective nonlinear model predictive control for a depression is the main contribution of this paper.

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Introduction

Abramson et al, studied various Cognitive vulnerability-stress models of depression in a self-regulatory and psychobiological context [1]. Hammen et al, studied the connection between stress and depression [2]. Griffioen-Both et al, modelled the dynamics of mood and depression [3]. Pittenger et al, investigated the relationship between Stress, depression, and neuroplasticity [4]. Demic et al, modelled the dynamics of disease states in depression [5]. Sheidow et al, investigated the role of stress exposure and family functioning in internalizing outcomes of urban families [6]. Hollon et al, studied the effect of cognitive therapy with antidepressant medications vs antidepressants alone on the rate of recovery in major depressive disorder [7]. Huang et al, used optimal control to disambiguate the effect of depression on sensorimotor, motivational, and goal-setting functions [8]. Farah et al, studied non-pharmacological strategies to treat depression [9]. Gartlehner and co-workers, investigated pharmacological and non-pharmacological treatments for major depressive disorders [10]. Vaishnaw performed a stability analysis of the mathematical models for depression in young women students [11].

Shah et al, discussed the curtailing of the severity of depression due to social pressure by medication [12]. Pedrelli et al, performed research about monitoring changes in depression severity using wearable and mobile sensors [13]. Best et al, developed mathematical models of serotonin, histamine, and depression [14]. Serotonin and the CNS. Tandon et al, studied the transmission dynamics of depression in socially connected populations [15]. Tosato et al, showed that Comt but not the 5httlpr gene is associated with depression in first-episode psychosis [16]. Guo et al, performed modeling and optimal control calculations of a new online game addiction model based on real data [17]. Ali et al, developed mathematical models, and performed analysis and numerical simulations of social media addiction and depression [18]. Nivetha et al, performed mathematical modeling and optimal control of depression dynamics influenced by saboteurs [19].

This paper aims to perform bifurcation analysis in conjunction with multiobjective nonlinear model predictive control (MNLMPC) for the depression dynamics model described in Nivetha et al, this paper is organized as follows [19]. First, the depression model equations are presented. The numerical procedures (bifurcation analysis and multiobjective nonlinear model predictive control (MNLMPC) are then described. This is followed by the results and discussion, and conclusions.

Depression Model Equations (Nivetha et al) [19].

The model parameter values are

$$\Lambda = 7.816; \lambda_1 = 0.2415; \lambda_2 = 0.7; \psi_1 = 0.0045; \gamma_1 = 0.6; \sigma = 0.01; \omega = 0.005; \psi_2 = 0.02; \mu = 0.0125;$$
$$\beta_1 = 0.5665; \beta_2 = 0.1567; \gamma_2 = 0.9219;$$

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u1,u2 and u3 are the control parameters.

$[sval, bval, pval, dval, mval, cval, rval]$ represent the susceptible individuals; detractors or saboteurs, individuals in the early stage of depression, individuals in the more advanced stage of depression, individuals undergoing non-pharmaceutical treatments, individuals undergoing pharmaceutical treatments, and individuals who have successfully managed their depression.

The model equations are

$$\begin{aligned}
\frac{d(sval)}{dt} &= \Lambda - \frac{(\beta_1(bval)(sval))}{nval} - \frac{(\beta_2(dval)(sval))}{nval};\\
\frac{d(bval)}{dt} &= \frac{(\beta_1(bval)(sval))}{nval} - \mu(bval)\\
\frac{d(pval)}{dt} &= \frac{(\beta_2(dval)(sval))}{nval} - \frac{(\lambda_2(bval)(pval))}{nval} - (\lambda_2 + \mu + \psi_1 + u1)(pval)\\
\frac{d(dval)}{dt} &= \frac{(\lambda_2(bval)(pval))}{nval} + \lambda_2(pval) - \omega(rval) - (\sigma + \mu + \psi_2 + u2)(dval)\\
\frac{d(mval)}{dt} &= ((\psi_1 + u1)pval) - (\gamma_1 + \mu)mval\\
\frac{d(cval)}{dt} &= ((\psi_2 + u2)dval) - (\gamma_2 + \mu + u3)cval\\
\frac{d(rval)}{dt} &= (\gamma_1)mval + (\gamma_2 + u3)cval + (\omega + \mu)rval
\end{aligned} \quad (1)$$

Bifurcation Analysis

The MATLAB software MATCONT is used to perform the bifurcation calculations. Bifurcation analysis deals with multiple steady-states and limit cycles. Multiple steady states occur because of the existence of branch and limit points. Hopf bifurcation points cause limit cycles. A commonly used MATLAB program that locates limit points, branch points, and Hopf bifurcation points is MATCONT (Dhooge Govearts, and Kuznetsov, 2003; Dhooge Govearts, Kuznetsov, Mestrom and Riet, 2004) [20,21]. This program detects Limit points (LP), branch points (BP) and Hopf bifurcation points (H) for an ODE system

$$\frac{dx}{dt} = f(x,\alpha)$$

$x \in R^n$ Let the bifurcation parameter be Since the gradient is orthogonal to the tangent vector,

The tangent plane at any point $w = [w_1, w_2, w_3, w_4, ...w_{n+1}]$ must satisfy

$$Aw = 0 \quad (3)$$

Where A is

$$A = [\partial f / \partial x \quad | \partial f / \partial \alpha] \quad (4)$$

where $\partial f / \partial x$ is the Jacobian matrix. For both limit and branch points, the matrix $[\partial f / \partial x]$ must be singular. The n+1 th component of the tangent vector = 0 for a limit point (LP)and for a branch point (BP) the matrix $\begin{bmatrix} A \\ w^T \end{bmatrix}$ must be singular. At a Hopf bifurcation point,

$$\det(2 f_x(x,\alpha) @ I_n) = 0 \quad (5)$$

@ indicates the bialternate product while I_n is the n-square identity matrix. Hopf bifurcations cause limit cycles and should be eliminated because limit cycles make optimization and control tasks very difficult. More details can be found in Kuznetsov and Govaerts [22-24].

Multiobjective Nonlinear Model Predictive Control (MNLMPC)

Flores Tlacuahuaz et al, developed a multiobjective nonlinear model predictive control (MNLMPC) method that is rigorous and does not involve weighting functions or additional constraints [25]. This procedure is used for performing the MNLMPC calculations Here $\sum_{t_i=0}^{t_i=t_f} q_j(t_i)$ (j=1, 2..n) represents the variables that need to be minimized/maximized simultaneously lem involving a set of ODE

$$\frac{dx}{dt} = F(x,u) \quad (6)$$

t_f being the final time value, and n the total number of objective variables and . u the control parameter. This MNLMPC procedure first solves the single objective optimal control problem independently optimizing each of the variables $\sum_{t_i=0}^{t_i=t_f} q_j(t_i)$ individually. The minimization/maximization of $\sum_{t_i=0}^{t_i=t_f} q_j(t_i)$ will lead to the values q_j^*. Then the optimization problem that will be solved is

$$\min(\sum_{j=1}^{n}(\sum_{t_i=0}^{t_i=t_f} q_j(t_i)-q_j^*))^2 \quad (7)$$
$$subject\ to\ \frac{dx}{dt}=F(x,u);$$

This will provide the values of u at various times. The first obtained control value of u is implemented and the rest are discarded. This procedure is repeated until the implemented and the first obtained control values are the same or if the Utopia point where

($\sum_{t_i=0}^{t_i=t_f} q_j(t_i)=q_j^*$ for all j) is obtained.

Pyomo (Hart et al, 2017) is used for these calculations [26]. Here, the differential equations are converted to a Nonlinear Program (NLP) using the orthogonal collocation method The NLP is solved using IPOPT (Wächter And Biegler, 2006) and confirmed as a global solution with BARON (Tawarmalani, M. and N. V. Sahinidis 2005) [27,28].

The steps of the algorithm are as follows

- Optimize $\sum_{t_i=0}^{t_i=t_f} q_j(t_i)$ and obtain at various time intervals ti. The subscript i is the index for each time step.
- Minimize $(\sum_{j=1}^{n}(\sum_{t_i=0}^{t_i=t_f} q_j(t_i)-q_j^*))^2$ and get the control values for various times.
- Implement the first obtained control values
- Repeat steps 1 to 3 until there is an insignificant difference between the implemented and the first obtained value of the control variables or if the Utopia point is achieved. The Utopia point is when $\sum_{t_i=0}^{t_i=t_f} q_j(t_i)=q_j^*$ for all j.

Sridhar (2024) proved that the MNLMPC calculations to converge to the Utopia solution when the bifurcation analysis revealed the presence of limit and branch points [29]. This was done by imposing the singularity condition on the co-state equation (Upreti, 2013) [30]. If the minimization of q_1 lead to the value q_1^* and the minimization of q_2 lead to the value q_2^* The MNLPMC calculations will minimize the function $(q_1-q_1^*)^2+(q_2-q_2^*)^2$. The multiobjective optimal control problem is

$$\min \quad (q_1-q_1^*)^2+(q_2-q_2^*)^2 \quad subject\ to\ \frac{dx}{dt}=F(x,u) \quad (8)$$

Differentiating the objective function results in

$$\frac{d}{dx_i}((q_1-q_1^*)^2+(q_2-q_2^*)^2)=2(q_1-q_1^*)\frac{d}{dx_i}(q_1-q_1^*)+2(q_2-q_2^*)\frac{d}{dx_i}(q_2-q_2^*) \quad (9)$$

The Utopia point requires that both $(q_1-q_1^*)$and $(q_2-q_2^*)$ are zero.

Hence

$$\frac{d}{dx_i}((q_1-q_1^*)^2+(q_2-q_2^*)^2)=0 \quad (10)$$

the optimal control co-state equation (Upreti; 2013) [30] is

$$\frac{d}{dt}(\lambda_i)=-\frac{d}{dx_i}((q_1-q_1^*)^2+(q_2-q_2^*)^2)-f_x\lambda_i; \quad \lambda_i(t_f)=0 \quad (11)$$

λ_i is the Lagrangian multiplier. t_f is the final time. The first term in this equation is 0 and hence

$$\frac{d}{dt}(\lambda_i)=-f_x\lambda_i;\lambda_i(t_f)=0 \quad (12)$$

At a limit or a branch point, for the set of ODE $\frac{dx}{dt}=f(x,u)$ f_x is singular. Hence there are two different vectors-values for $[\lambda_i]$ where $\frac{d}{dt}(\lambda_i)>0$. This coupled with the boundary condition $\lambda_i(t_f)=0$ will lead to $[\lambda_i]=0$ This makes the problem an unconstrained optimization problem, and the only solution

Results and Discussion

For the bifurcation analysis, β_1 is the bifurcation parameter a branch point occurred at $[sval, bval, pval, dval, mval, cval, rval, \beta_1]$ values of = (145.48, 0, 10.9, 207.38, 0.080, 4.438, 236.579, 0.05197). This is shown in Figure 1.

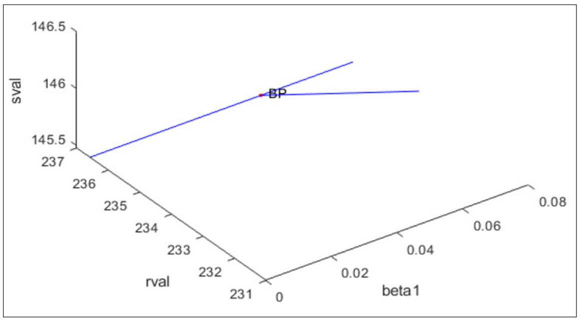

Figure 1: Bifurcation Analysis Indicating Branch Point

For the MNLMPC calculations,

$\sum_{t_i=0}^{t_i=t_f} dval(t_i), \sum_{t_i=0}^{t_i=t_f} cval(t_i), \sum_{t_i=0}^{t_i=t_f} pval(t_i); \sum_{t_i=0}^{t_i=t_f} mval(t_i)$ were minimized

individually and each led to a value of 0. The multiobjective optimal control problem will involve the minimization of

$(\sum_{t_i=0}^{t_i=t_f} dval(t_i)-0)^2+(\sum_{t_i=0}^{t_i=t_f} cval(t_i)-0)^2+(\sum_{t_i=0}^{t_i=t_f} pval(t_i)-0)^2+(\sum_{t_i=0}^{t_i=t_f} mval(t_i)-0)^2$ subject to the equations governing the model. This led to a value of zero (the Utopia solution). The MNLMPC control values obtained for u1, u2 and u3 are 0.8026; 1.378; 0.9478

The various profiles for this MNLMPC calculation are shown in Figures 2,3,4. The obtained control profile of u1 and u2 u3 exhibited noise (Figure 5). This issue was addressed using the Savitzky-Golay Filter. The smoothed version of this profile is shown in Figure 6. The MNLMPC calculations converged to the Utopia solution, validating the analysis by Sridhar (2024), which demonstrated that the presence of a limit point/branch point enables the MNLMPC calculations to reach the optimal (Utopia) solution.

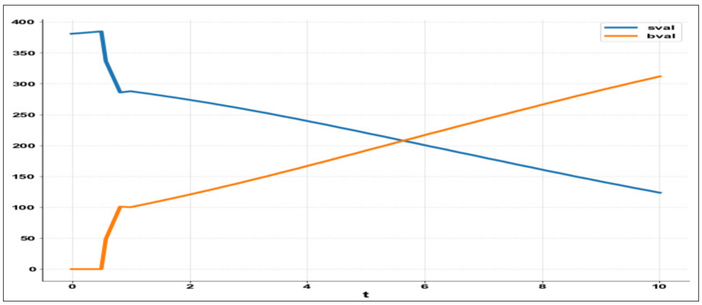

Figure 2: MNLMPC sval, bval Profiles

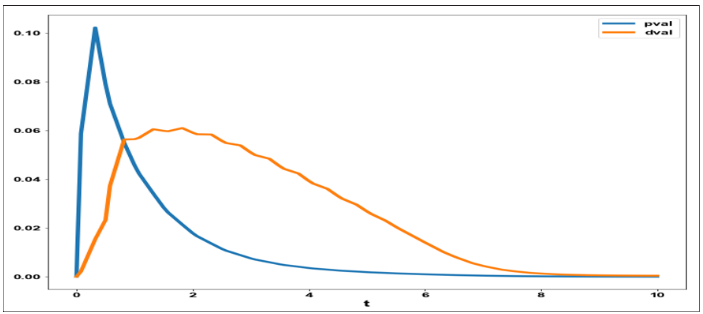

Figure 3: MNLMPC dval, bval Profiles

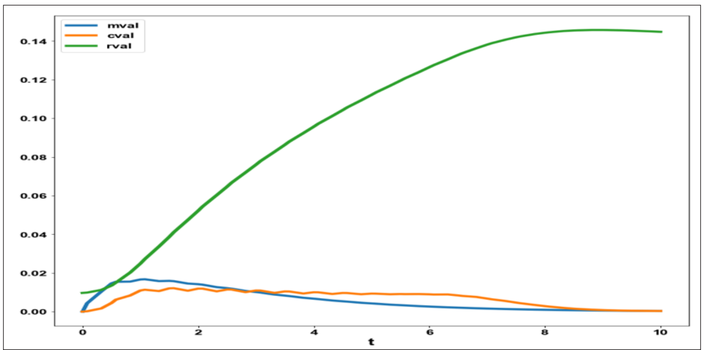

Figure 4: MNLMPC mval, cval, rval Profiles

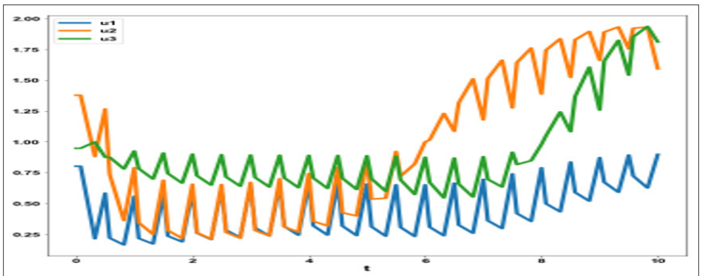

Figure 5: MNLMPC u1 u2 u3 Profiles (Noisy)

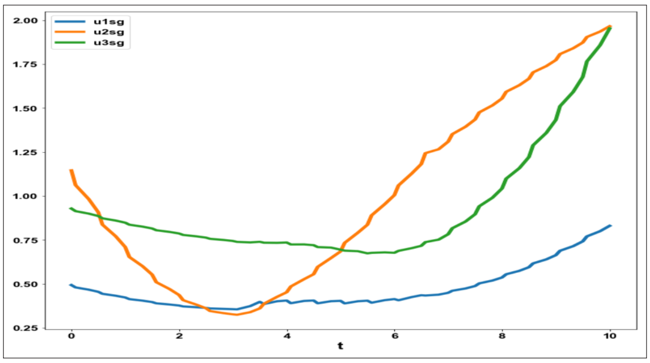

Figure 6: MNLMPC u1, u2, u3 Profiles (Noise Eliminated with Savitzky Golay Filter)

Conclusions

Bifurcation analysis and Multiobjective nonlinear model predictive control calculations were performed on a dynamic depression model. The bifurcation analysis revealed the existence of a branch point. The branch point (which causes multiple steady-state solutions from a singular point) is very beneficial because it enables the Multiobjective nonlinear model predictive control calculations to converge to the Utopia point (the best possible solution) in this model. A combination of bifurcation analysis and multiobjective nonlinear model predictive control for a dynamic depression models is the main contribution of this paper.

Data Availability Statement

All data used is presented in the paper

Conflict of Interest

The author, Dr. Lakshmi N Sridhar has no conflict of interest.

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